

Measurements of spanwise scale change in a forced mixing layer

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Spanwise scale changes of the streamwise vortical structure in a plane forced mixing layer have been investigated through direct measurements. Detailed three-dimensional phase-averaged measurements were obtained of the spanwise and streamwise vorticity in a forced mixing layer undergoing three spanwise roller pairings. A two-stream mixing layer with a velocity ratio (U_2/U_1) of 0.6 and laminar initial boundary layers was generated in a mixing-layer wind tunnel. Acoustic forcing, consisting of a fundamental roll-up frequency and its first, second and third subharmonics, was used to phase-lock the initial development and the first three pairings of the spanwise rollers. Although the overall spanwise scale remained unchanged through the first two roller pairings, some (cyclic) ‘readjustment’ of the weaker streamwise structures was observed. The overall spanwise scale doubled during the third roller pairing. For the first time, one of the proposed mechanisms for the scale change has been identified and its details measured directly. The weakest (positive) streamwise vortex is split into two and displaced by stronger neighbouring (negative) vortices. These two vortices (of the same sign) then merge together, thus doubling the spanwise scale and circulation of the resulting streamwise vortical structure.

1. Introduction

The three-dimensional structure of plane transitioning mixing layers has been the subject of many experimental and computational studies since the early 1970s when it was realized that, in addition to the familiar spanwise vortices (‘rollers’), a secondary structure was also generated. The secondary structure took the form of ‘spatially-stationary’ streamwise vortices which were soon identified in flow visualization studies (Konrad 1976; Breidenthal 1981; Jimenez, Cogollos & Bernal 1985; Bernal & Roshko 1986; Lasheras, Cho & Maxworthy 1986) and in velocity measurements (Jimenez 1983). These earlier results showed that the streamwise structures (‘ribs’) first formed in the *braid* region, a region connecting adjacent spanwise vortices, and that their locations were related to the strength and position of (weak) incoming disturbances.

There was some disagreement regarding the subsequent evolution of the streamwise structures. The results of Breidenthal (1981), Lasheras *et al.* (1986), and Lasheras & Choi (1988) showed no change in the spanwise spacing or wavelength of the streamwise structures with downstream distance. In contrast, Konrad (1976), Jimenez (1983), and Jimenez *et al.* (1985) found that the spanwise wavelength increased in a stepwise

fashion. Specifically, the three-dimensional reconstructed models of Jimenez *et al.* (1985) showed that the spanwise wavelength doubled during a pairing of the rollers. Jimenez (1983) suggested that the observed increase in wavelength may be due to annihilation of neighbouring (opposite sign) rib vortices, thus leaving ribs with twice the original spacing. The flow visualization studies of Bernal & Roshko (1986) did not show a spanwise scale change through the first couple of roller pairings, but then after completion of the so-called 'mixing transition', it increased linearly. Both Jimenez and Bernal & Roshko reported that the overall increase in spanwise wavelength scaled approximately with the mixing-layer vorticity thickness.

Huang & Ho (1990) investigated the adjustment of the spanwise scale in a two-stream mixing layer, both with and without two-dimensional acoustic forcing. Based on the maximum entropy spectra of their spanwise velocity distributions, they concluded that the spanwise wavelength of the streamwise structures doubled after each of the first two pairings of the spanwise rollers that were investigated. However, the peak from the initial (shorter) wavelength was apparent even after the second spanwise roller pairing, although its magnitude was much smaller than that of the new longer wavelength. Huang & Ho suggested that this persistence of the shorter wavelength was perhaps responsible for the constant wavelength observed for relatively long streamwise distances in the flow visualization studies of Konrad (1976) and Bernal & Roshko (1986). Huang & Ho (1990) also reported that the ratio of spanwise to streamwise wavelength was maintained at a value of 0.67, which corresponds to the most amplified wavelength according to stability analyses (Pierrehumbert & Widnall 1982; Rogers & Moser 1993).

Bell & Mehta (1992) studied the initial development and far-field evolution of the streamwise vortical structure through detailed time-averaged measurements. The mean vorticity first appeared in 'clusters' containing vorticity of both signs, but further downstream, it 're-aligned' to form counter-rotating pairs in a nominally linear arrangement. The vortex structure was then found to grow in size with downstream distance, and the spanwise wavelength associated with them scaled approximately with the local mixing-layer vorticity thickness, in agreement with the earlier observations of Jimenez (1983) and Bernal & Roshko (1986). Furthermore, the increase in this study also exhibited a stepwise behaviour with the jumps in spanwise wavelength coinciding approximately with the estimated locations of roller pairings. However, a jump in spanwise wavelength was not observed at every estimated roller pairing location and the spanwise wavelength remained constant over fairly large streamwise distances. This was proposed as another reason why Breidenthal (1981) and Lasheras *et al.* (1986) did not observe a change in spacing. Since the jumps in spanwise scale were correlated with local increases in the average streamwise circulation per vortex, Bell & Mehta (1992) suggested that at least one mechanism for the increase in spanwise scale was amalgamation of vortices of the same sign.

Tung (1992) measured all three velocity components in a forced mixing layer and used Taylor's hypothesis to transform (phase-averaged) temporal measurements onto a 'spatial' domain. Tung first observed concentrated streamwise vortices at the start of the first spanwise structure pairing, and these, together with additional generated vortices, soon formed the familiar single row of counter-rotating pairs. Toward completion of the first roller merging, he also observed a pairing between streamwise vortices of the same sign, as had been previously hypothesized by Bell & Mehta (1992). However, a global doubling of the spanwise wavelength was not observed downstream of the roller pairing. Tung's ratio of spanwise to streamwise wavelength was also approximately 0.67.

LeBoeuf & Mehta (1995*a*) recently demonstrated that Taylor's hypothesis is not suitable for the study of three-dimensional mixing layers. They therefore proceeded to obtain three-dimensional phase-averaged measurements of the spanwise and streamwise vorticity in a forced mixing layer without invoking Taylor's hypothesis (LeBoeuf & Mehta 1995*b*). Acoustic forcing, consisting of a fundamental roll-up frequency and its subharmonic, was used to phase-lock the initial development and first pairing of the spanwise vortical structures. The results showed that the streamwise vorticity first appeared in the form of ribs just upstream of the first spanwise vortex roll-up. At the same time, the first spanwise roller became kinked, thus also contributing to the streamwise vorticity. The morphology of the surviving rib vortices was not really affected by the spanwise vortex pairing, but the relative contribution due to kinking of the spanwise rollers was reduced further downstream, thus explaining the 're-alignment' of the streamwise structure distribution reported by Bell & Mehta (1992). In particular, the spanwise wavelength of the streamwise structures did not increase within the measurement domain. Consequently, the spanwise to streamwise wavelength ratio decreased from 1.3 upstream of the roller pairing to 0.65 downstream of it.

The issue of spanwise scale change has also been addressed using direct numerical simulations. Rogers & Moser (1993) conducted simulations of temporally evolving mixing layers, starting from the formation of the initial spanwise vortex roll-up up to the third roller pairing. With infinitesimal initial three-dimensional disturbances, it was found that three or more roller pairings were required to complete a doubling of the spanwise scale. More rapid scale changes could be produced with stronger three-dimensionality, but this also triggered early transition to turbulence. Depending on the amplitudes of the initial disturbances, a doubling (or even quadrupling) of the spanwise scale could be obtained after zero, one, or two roller pairings. Their results therefore showed that the details of the spanwise scale change are dependent on the details of the disturbance environment, thereby making the whole process 'facility dependent'. Two mechanisms were also identified by which an array of counter-rotating rib vortices may reorganize into another array with enlarged lengthscale. In the first mechanism, a roughly equal strength rib pair is brought together by the induced motion of the neighbouring ribs and is eventually viscously annihilated. This is similar to the mechanism proposed by Jimenez (1983). In the second mechanism, a single strong rib can induce nearby weaker ribs of opposite sign to rotate around it and viscously amalgamate into a single weaker vortex.

Although the issue of spanwise scale change of the streamwise vortical structures in mixing layers has already received considerable attention, there is still some confusion over when and how such a scale change will occur in practical mixing layers. In particular, details of the streamwise structure scale change have not been studied quantitatively in experiments. In the present study, acoustic forcing was used to phase-lock the formation and first three pairings of the spanwise rollers, since a scale change was not observed through the first pairing in our earlier study (LeBoeuf & Mehta 1995*b*). Phase-averaged spanwise and streamwise vorticity measurements were used to identify the presence and form of the spanwise scale change. The mechanism responsible for the observed scale change has also been identified.

2. Experimental apparatus and techniques

The experimental apparatus was essentially the same as that described in LeBoeuf & Mehta (1995*b*); only a brief description is therefore given here. The experiments

were conducted in a mixing-layer wind tunnel with a test section which is 36 cm in the cross-stream direction, 91 cm in the spanwise direction, and 366 cm in length. An adjustable sidewall is used to zero the streamwise pressure gradient. To facilitate three-dimensional traversing, a slotted aluminium plate was mounted on the traverse which moves the probe in the streamwise (X) direction.

In the present experiments, the two sides of the mixing layer were set to 12 m s^{-1} and 7.2 m s^{-1} giving a velocity ratio, $r = U_2/U_1 = 0.6$ [$\lambda = (U_1 - U_2)/(U_1 + U_2) = 0.25$], the same operating conditions as those used in our previous study (LeBoeuf & Mehta 1995*b*). At these operating conditions, the streamwise and transverse free-stream turbulence intensities were approximately 0.15% and 0.05%, respectively, and the boundary layers on the splitter plate were laminar and nominally two-dimensional.

Velocity measurements were made using a single cross-wire probe which was rotated in order to obtain data in two-coordinate planes (uv and uw). The cross-wire probe consisted of $5 \mu\text{m}$ platinum-plated tungsten sensing elements approximately 1 mm long with approximately 1 mm spacing. The probes were calibrated statically in the potential core of the flow assuming a 'cosine-law' response to yaw, with the effective wire angles determined by calibration. The analogue signals from the anemometers were sampled using a 15 bit simultaneous sample-and-hold A/D converter with a multiplexed connection to a computer.

Conversion of the hot-wire data to effective normal velocities was achieved online through the use of look-up tables. Corrections for the effects of the mean streamwise velocity gradient across the probe face were performed using quadratic interpolation of the effective velocity statistics to the actual probe location (centred between the wires). Further reduction of the measurements to vorticity was achieved using a central difference scheme with forward and backward differences at the grid boundaries. Circulation was evaluated using an integral over a bicubic spline fit of the vorticity for individual structures. An error analysis based upon repeatability and calibration accuracy indicated that the time-averaged mean streamwise and cross-stream velocities were accurate to within 2% and 7%, respectively. The phase-averaged streamwise velocity and cross-stream velocities were accurate to within 5% and 10%, respectively, and the phase-averaged vorticity measurements were accurate to within 15%.

A digital sine wave generator capable of outputting four signals simultaneously was built for this project. The forcing signals used to obtain the results shown in this study consisted of the sum of a sine wave at the fundamental 'most-probable' roll-up frequency (obtained from centreline spectra in the unforced layer), and its first, second and third subharmonics:

$$s(t) = A_1 \sin 2\pi f_1 t + A_2 \sin (2\pi f_2 t + \beta_2) + A_3 \sin (2\pi f_3 t + \beta_3) + A_4 \sin (2\pi f_4 t + \beta_4), \quad (1)$$

where $f_1 = 500 \text{ Hz}$, $f_2 = 250 \text{ Hz}$, $f_3 = 125 \text{ Hz}$ and $f_4 = 62.5 \text{ Hz}$. The amplitudes (A_n), which largely dictate the pairing location for multiple-frequency forcing, were set equal for all of the measurements described in this paper. This forced the first, second and third pairings to begin about 15, 30 and 60 cm downstream of the splitter-plate trailing edge, respectively. The forcing signal relative phase angles, β_2 , β_3 and β_4 , determined from acoustic measurements in the test section, were 20° , 0° and 30° , respectively. These values were the result of optimizing all three relative phases in order to induce rolling-type interactions for all three-pairings. The individual sine waves were combined using a simple summing circuit and output via an audio amplifier to a spanwise array of three four-inch speakers which were placed directly across from the splitter-plate trailing edge at a sidewall slot location. The acoustic signal at the slot was measured to be spanwise uniform to within 5%. The amplitude (volume) of the output

signal from the amplifier was set to the absolute minimum level (~ 70 dB) which still gave adequate coherence in the phase-locking. The measured periodic contribution to the total Reynolds stress (for the u and v) components was higher than 35% over most of the measurement domain.

The sine wave generator also output digital start sample pulses and a clock signal which were used to synchronize A/D sampling with the forcing signals. For the phase-averages shown in the next section, 768 ensembles of 32 samples per cycle were measured. The measurement grid consisted of 155 uniformly spaced X locations in the range 1 to 78 cm and 11 uniformly spaced Y locations distributed over a linearly increasing range of -1 to 1 cm at $X = 1$ cm to -6.2 to 4.4 cm at $X = 78$ cm. In the spanwise direction, the three-dimensional grid ranged from $Z = 0$ to 5 cm with 21 uniformly spaced locations. Note that this grid occupied approximately 5.5% of the total test-section span and $Z = 0$ corresponds to the test-section centreline.

3. Results and discussion

Following convention, the velocity scale used to normalize all the data presented in the present paper is the velocity difference across the mixing layer, $U_0 = U_1 - U_2 = 4.8 \text{ m s}^{-1}$.

The streamwise evolution of phase-averaged spanwise vorticity ($\langle \Omega_z \rangle / U_0 \text{ cm}^{-1}$) along the mixing-layer centreline ($Z = 0$) is depicted in figure 1 for eight phases (or times). In effect, every fourth phase out of the 32 measured phases per lowest (third) subharmonic cycle is presented. The evolution and pairings of sets of primary rollers can be easily tracked through the eight phases shown. The initial spanwise vortex roll-up occurs at $X \approx 5$ cm and the spanwise rollers are shed with an initial streamwise wavelength of about 1.75 cm. Clearly, signs of subharmonic forcing are present early in the mixing-layer development since distinguishable sets of primary rollers are discernible at the onset of their development. Spanwise vortices of each pair start to move closer together between $X \approx 10$ and 15 cm, begin to corotate between $X \approx 15$ and 20 cm, and complete the first pairing by $X \approx 25$ cm. The second pairing is initiated immediately after the first pairing is complete, beginning between $X \approx 25$ and 30 cm and ending at $X \approx 50$ cm. The third pairing, which follows immediately, remains in progress at the end of the measurement domain at $X = 78$ cm. The peak phase-averaged spanwise vorticity levels drop from 4 cm^{-1} to 1 cm^{-1} during the first pairing, down to 0.5 cm^{-1} during the second pairing and finally down to 0.4 cm^{-1} during the third pairing.

Note that at a given phase, structure-to-structure differences in the phase-averaged quantities characterize the changes induced in a structure as it progresses downstream during one cycle of the forcing signal. Furthermore, at a given streamwise location, progression in phase illustrates the state of a structure which was upstream at an earlier phase. Of course, if no evolution takes place as a structure moves downstream, progression in phase could be used to characterize spatial development which would mean that Taylor's 'frozen-field' hypothesis would be applicable. However, it is clearly apparent that even the qualitative features of individual structures change as the structures evolve spatially, especially if a large number of subharmonics are included in the forcing signal. In fact, relatively large errors are introduced when employing Taylor's hypothesis to evaluate vorticity development in three-dimensional mixing layers as recently demonstrated by LeBoeuf & Mehta (1995*a*).

Since the objective of the present study was to investigate spanwise scale changes associated with the rib structures, we decided to follow the spatial evolution of

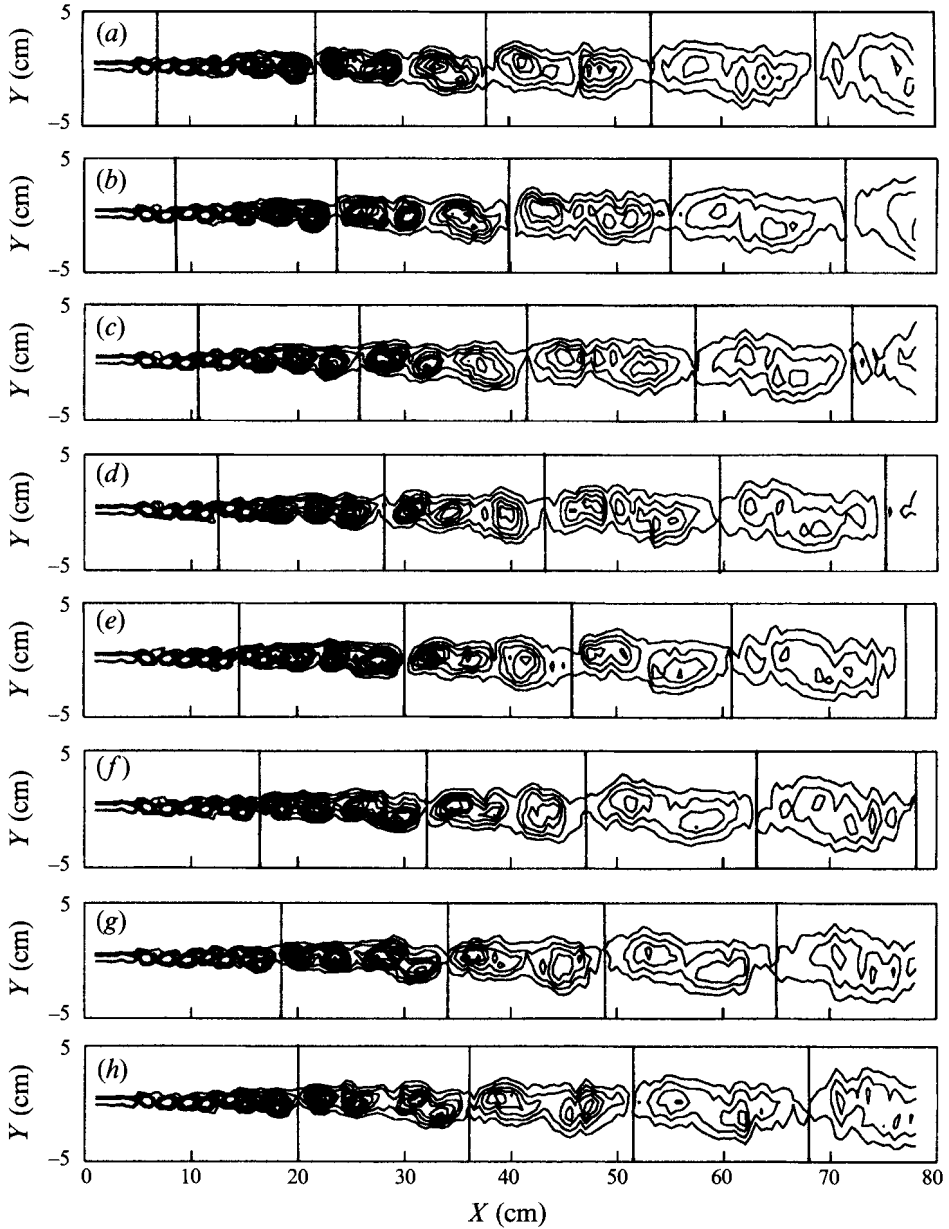


FIGURE 1. Centreplane phase-averaged spanwise vorticity ($\langle\langle\Omega_z\rangle\rangle/U_0$, cm^{-1}) contours at various phases. Vertical lines mark the location of the surviving braid region tracked in figure 2. Lowest contour level = -0.113 , increment = -0.1 . (a) Phase 1; (b) phase 5; (c) phase 9; (d) phase 13; (e) phase 17; (f) phase 21; (g) phase 25; (h) phase 29.

surviving rib structures as they evolved through the three spanwise roller pairings. This is achieved by marching in X and phase at the same time. With 32 phases per third subharmonic forcing cycle ($1/62.5 \text{ Hz} = 0.016 \text{ s}$ duration), this gives a streamwise evolution of 0.48 cm per phase, based on an average convection velocity, $\frac{1}{2}(U_1 + U_2) = 9.6 \text{ m s}^{-1}$. Of course, as adjacent rollers pair, the braid region between them is engulfed into the new roller. Therefore, the so-called 'surviving' braid region was chosen for examination of its streamwise evolution through the three roller

pairings. The surviving braid region selected for the present scrutiny is marked by the first vertical line in figure 1(a). The location of that particular surviving braid region can then be followed by tracking the streamwise displacement of the vertical line through the eight phases. Of course, to continue tracking the surviving braid region beyond the location indicated by a line in figure 1(h), it is necessary to continue with the next downstream location indicated in figure 1(a), and so on.

In order to investigate the spanwise scale of the rib structures, cross-stream (Y, Z) plane cuts through the selected surviving braid region are presented in figure 2 as this region evolves through the first, second and third primary roller pairings. At $X = 7$ cm (figure 2a), approximately two pairs of streamwise vortex pairs are apparent, with the stronger pair on the left ($0 < Z < 2.5$ cm) and a weaker, less organized one on the right ($2.5 < Z < 5$ cm). In particular, the positive vortex at $Z \approx 3$ cm appears to be split into two smaller structures and the negative one at $Z \approx 4.5$ cm is considerably weaker than its counterpart at $Z \approx 2$ cm. The average spacing between the streamwise vortices is about 1.5 cm which gives an initial spanwise to streamwise wavelength ratio of about 1.7. Further downstream, by $X = 14.5$ cm (figure 2c), the two weak positive structures have merged into one and the weak negative structure appears to have merged with the negative vorticity above it. The new positive vorticity at $Z \approx 4.5$ cm seems to have been pulled into the measurement domain from the right. Somewhat surprisingly, at $X = 18$ cm (figure 2e), the weak positive structure starts to split up again and the negative region above it has broken away from the stronger negative structure. Figure 1(g) shows that at this location and phase, the selected braid region is right in the middle of the first roller pairing process. The upstream pair is in the process of rolling-up while the downstream pair has almost completely merged together.

Further downstream, the split positive pair merges together again and by $X = 25$ cm, at a time (phase) at which the braid is located between two paired rollers, nearly two complete pairs of streamwise vortices are apparent (figure 2i). So while there is no scale change in the region of the first roller pairing, as such, the weaker vortices do undergo some temporary changes in structure and/or position. A similar behaviour of the weaker vortices in this region was also observed in our previous study (LeBoeuf & Mehta 1995b). It is interesting to note that the streamwise vorticity level of the positive rib of the strong pair decreased by about a factor of two while that of the weaker negative rib in the stronger pair, as well as the streamwise vorticity of the weaker rib pair, was more or less maintained throughout the first pairing. Using measurements on a larger domain, LeBoeuf & Mehta (1995b) also found that the surviving rib streamwise vorticity averaged over each individual identified rib did not change significantly through the first pairing.

The trends observed in the region of the first pairing continue during the second pairing. Following the weak positive vortex at $Z \approx 3$ cm, between $X = 25$ cm (figure 2i) and $X = 26.5$ cm (figure 2j), the vortex splits again into two structures. Referring to figure 1(c), this first split takes place at the onset of the second pairing which is occurring downstream of the surviving braid region. The structure recombines by $X = 32.5$ cm (figure 2k), when the downstream second pairing is approaching completion and at the onset of the upstream second pairing (see figure 1g). The structure splits again during the upstream pairing (see figure 1h) at $X = 34.5$ cm (figure 2l). Recombination occurs by $X = 39$ cm (figure 2m) at which time the upstream second pairing is approaching its completion as seen in figure 1(b). The weak vortex at $Z \approx 3$ cm splits and recombines again between $X = 44.5$ and 46 cm (figures 2n and 2o) during the beginning of the third pairing. Note that, throughout its evolution through the first and second pairing, the stronger pair on the left ($0 < Z < 2.5$ cm) does

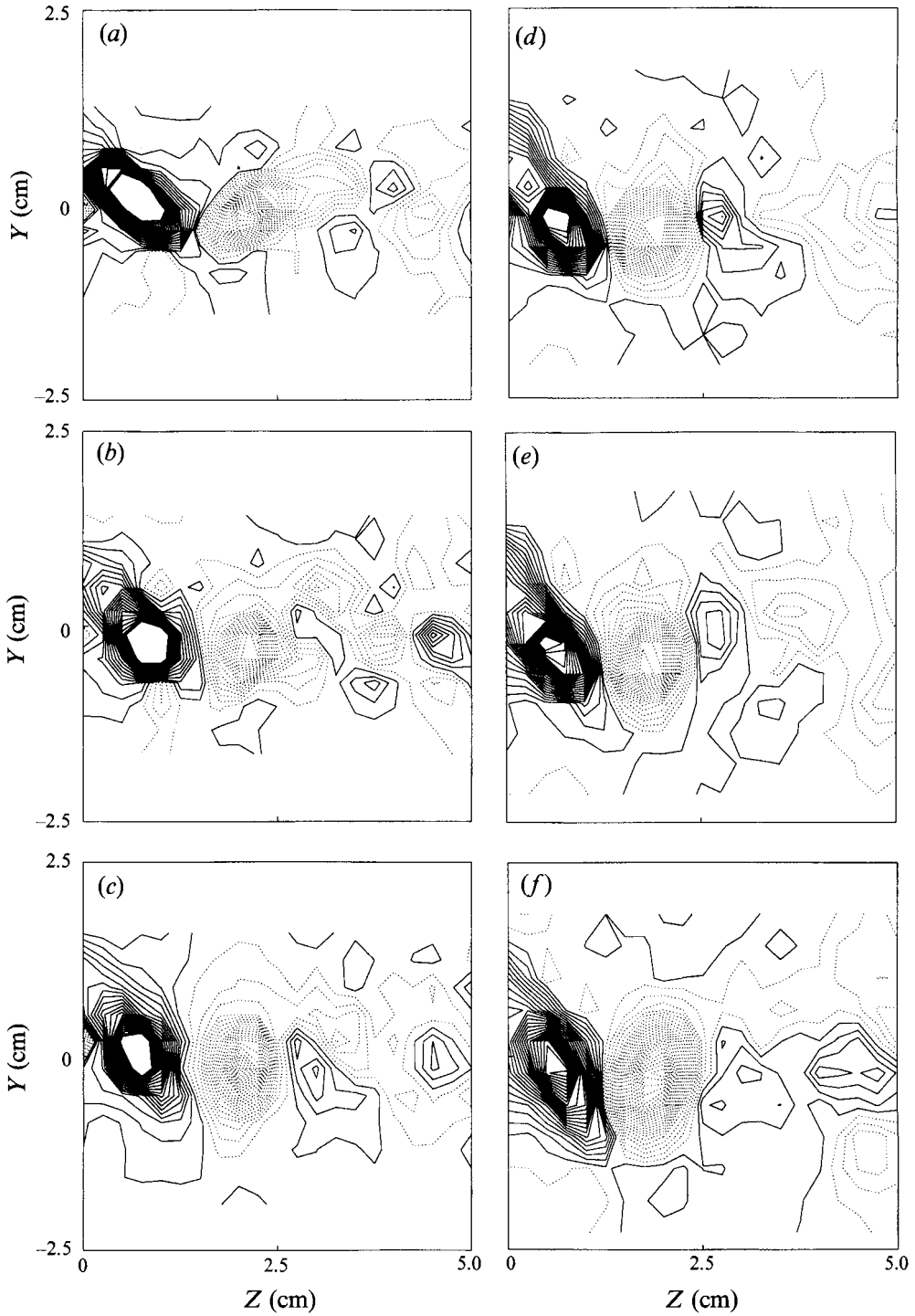


FIGURE 2(a-f). For caption see p. 316.

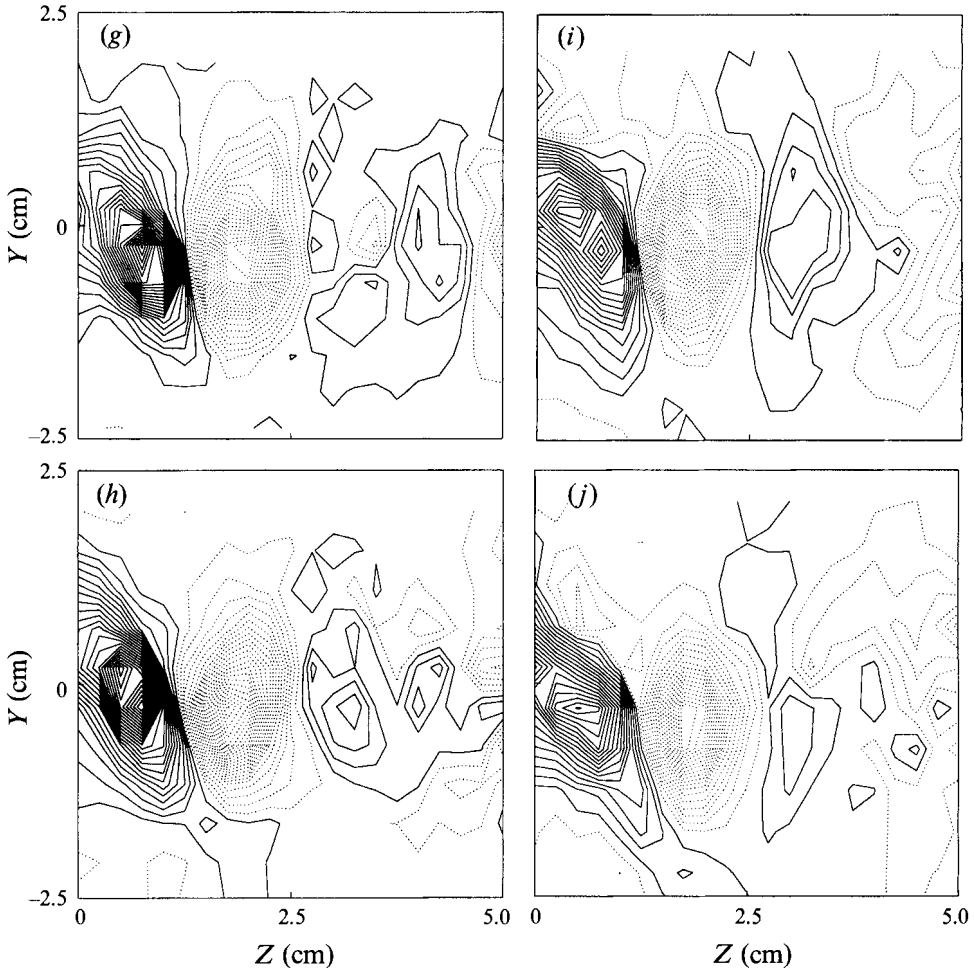


FIGURE 2(g-j). For caption see p. 316.

not change much, except that the vorticity levels drop with streamwise distance as the vortices diffuse and thus enlarge. However, the enlargement only takes place in the vertical (Y) direction; the spanwise growth is presumably restricted by the neighbouring streamwise vortices. In fact, the spacing between the two vortices does not change at all in the region from $X = 6.5$ to 46 cm (figure 2*a-o*).

By $X = 46.5$ cm (figure 2*p*), the weak positive vortex at $Z \approx 3$ cm splits again, but this time the two neighbouring negative vortices move closer together into the area vacated by the weak positive vortex and start to amalgamate. The fragments of the shredded positive vortex appear to move out of the measurement domain. At $X = 57$ cm (figure 2*t*), only a single pair of streamwise vortices is clearly visible with the distance between the pair doubled. This single pair persists to the end of the measurement domain ($X = 76$ cm, figure 2*v*). The spacing between the pair is about 3.6 cm which gives a spanwise to streamwise wavelength ratio of about 0.5 .

Clearly the mechanism for the increase in spanwise scale in the present study is that the weakest (positive) vortex splits up and moves out of the array, the two neighbouring vortices (both negative) move closer together and amalgamate and the spanwise spacing is hence readjusted. Apparently there was a tendency for the weakest

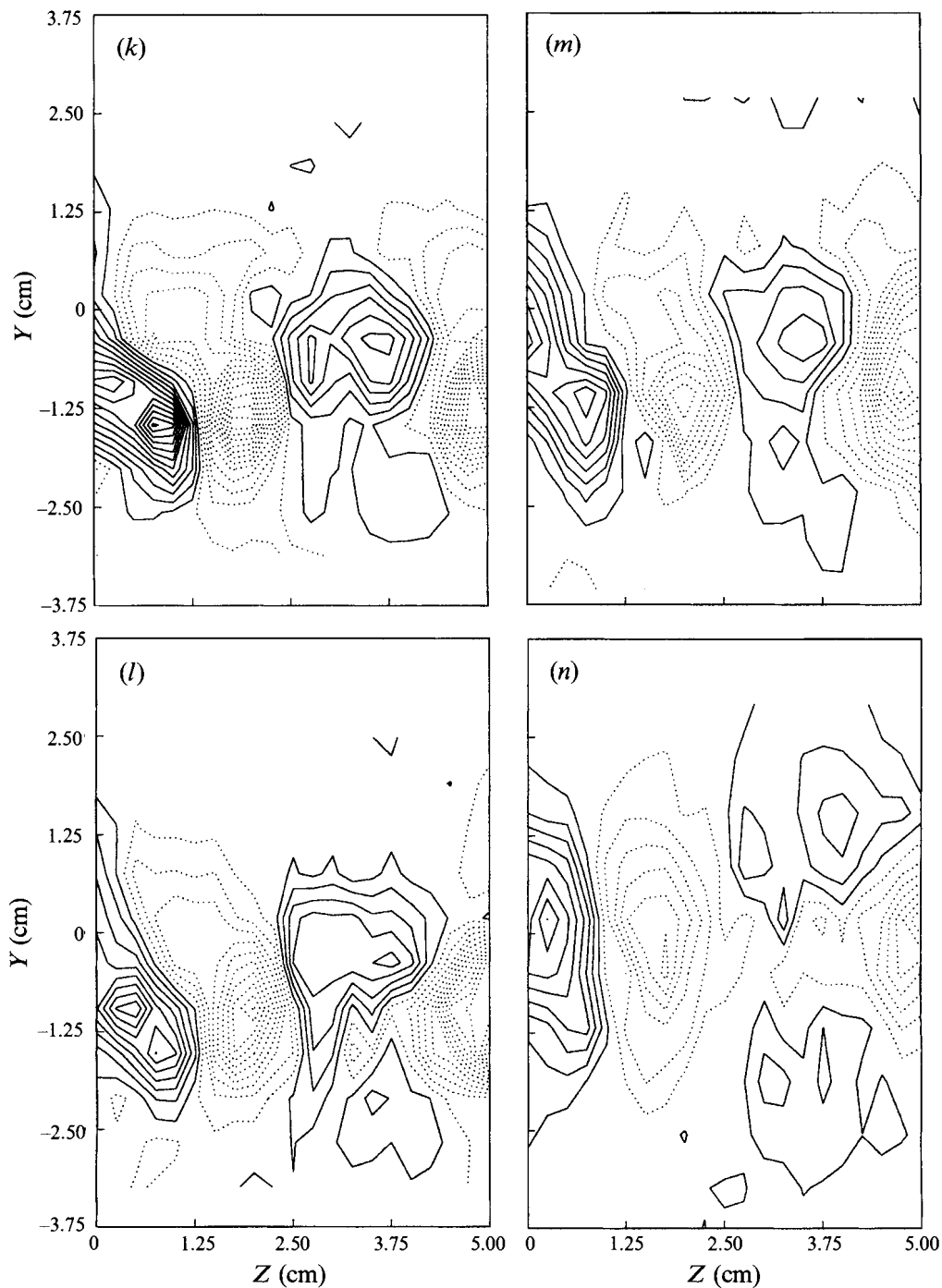


FIGURE 2(k-n). For caption see p. 316.

vortex to split and thereby permit merging of the two like-signed adjacent vortices during both the first and second pairing, however, the merging finally occurred during the third primary roller pairing. If this was a true amalgamation of two vortices of the same sign, then one would expect the circulation of the new (merged) vortex to be

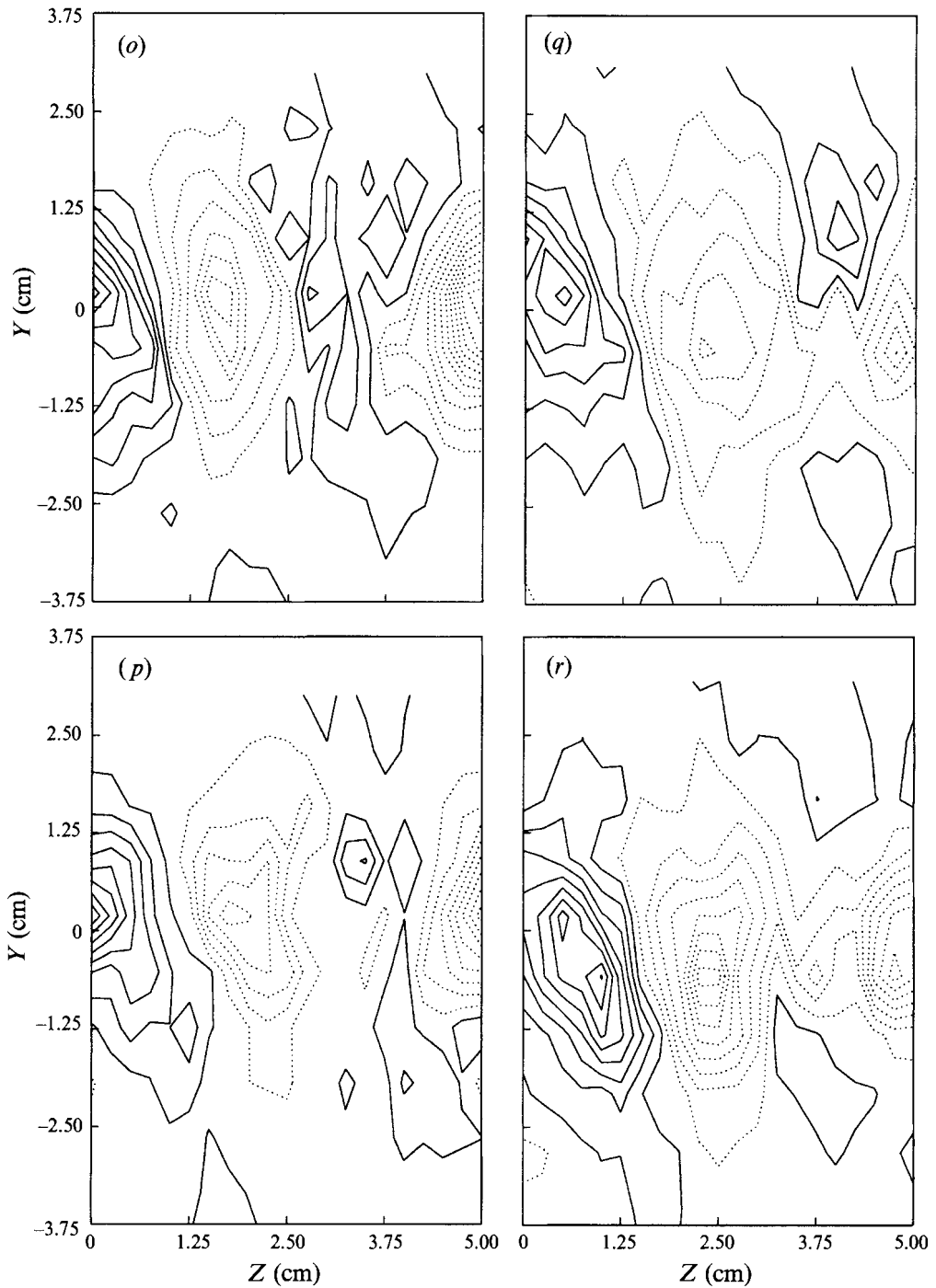


FIGURE 2(o-r). For caption see p. 316.

higher than those of the original vortical structures. And indeed, in the present results, the circulation of the merged negative structure in figure 2(t) is twice that of one of the original negative structures (that at $Z \approx 2$ cm in figures 2o and 2p).

Two-dimensional forcing is known to have a profound effect on the formation and

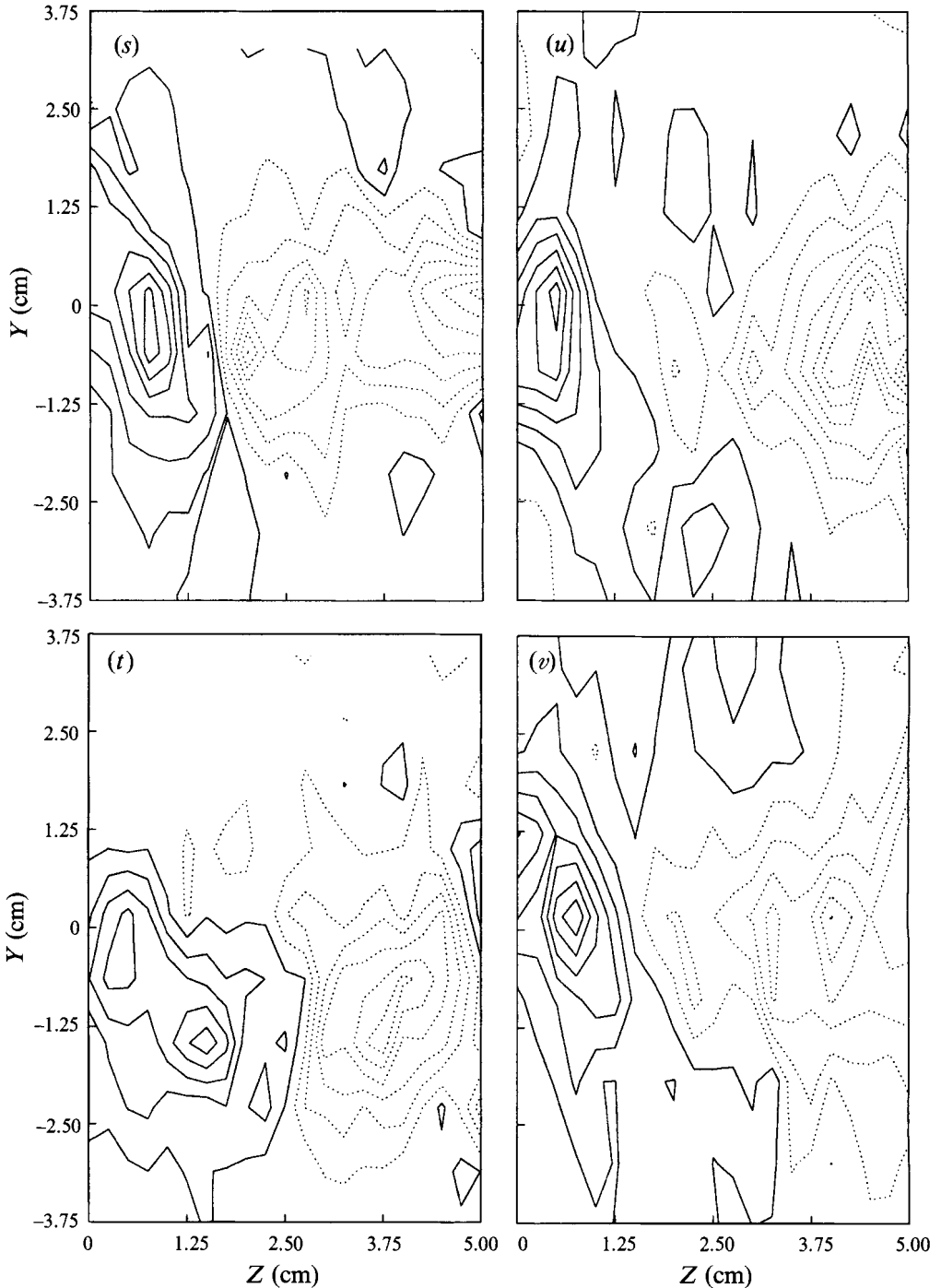


FIGURE 2. Cross-stream plane phase-averaged streamwise vorticity ($\langle \Omega_x \rangle / U_0$, cm^{-1}) contours. \dots , negative; $-$, positive; lowest level = ± 0.013 , increment = ± 0.025 . (a) $X = 7$ cm, phase 1; (b) $X = 10.5$ cm, phase 9; (c) $X = 14.5$ cm, phase 17; (d) $X = 17$ cm, phase 23; (e) $X = 18$ cm, phase 25; (f) $X = 20$ cm, phase 29; (g) $X = 21.5$ cm, phase 32; (h) $X = 24$ cm, phase 5; (i) $X = 25$ cm, phase 7; (j) $X = 26.5$ cm, phase 11; (k) $X = 32.5$ cm, phase 24; (l) $X = 34.5$ cm, phase 28; (m) $X = 39$ cm, phase 5; (n) $X = 44.5$ cm, phase 14; (o) $X = 46$ cm, phase 17; (p) $X = 46.5$ cm, phase 18; (q) $X = 48.5$ cm, phase 23; (r) $X = 50.5$ cm, phase 27; (s) $X = 52.5$ cm, phase 31; (t) $X = 57$ cm, phase 9; (u) $X = 73$ cm, phase 9; (v) $X = 78$ cm, phase 19.

interaction of the spanwise vortical structures (Ho & Huerre 1984). The development of the peak mean streamwise vorticity was also recently found to be affected somewhat by two-dimensional forcing (LeBoeuf & Mehta 1995*b*). In order to confirm that the spanwise scale change observed here was not an artifact of the imposed acoustic forcing, time-averaged streamwise vorticity measurements for the forced mixing layers (averaged over all 32 phases) were compared to those obtained in the same mixing layer, but with the forcing turned off. All other initial and operating conditions were maintained exactly the same in this unforced mixing layer. In both cases, the spanwise scale, as determined from the mean streamwise vorticity measurements, was constant out to $X \approx 50$ cm, after which it doubled abruptly. Therefore, the forcing is obviously not responsible for at least the presence and location of the observed scale change. Huang & Ho (1990) also showed that the streamwise development of the three-dimensional (spanwise) to two-dimensional (streamwise) wavelength ratio was not affected by their acoustic forcing.

The present results agree, in particular, with many of the observations and notions of Bell & Mehta (1992), developed from their time-averaged measurements obtained in an unforced mixing layer. They also observed the first rapid scale change at $X \approx 50$ cm, which was estimated to be the location of the third roller pairing. As in the present study, the spanwise scale was found to double in this region, only over a much wider spanwise extent – that including six streamwise vortex pairs instead of the two covered in the present investigation. Bell & Mehta's results also showed that the spanwise scale was relatively constant in the region between the rapid increases or jumps. These observations imply that, for the initial conditions in our facility, the spanwise scale change occurs at several spanwise locations and at the same time (i.e. during a given roller pairing).

Since the locations of the scale changes in Bell & Mehta's (1992) results were correlated with local increases in the average streamwise circulation per vortex, they suggested that the mechanism for the increase in spanwise scale was amalgamation of vortices of the same sign, which is exactly what is observed here. Tung (1992) also reported seeing some (local) pairings of streamwise vortices of the same sign. While the other proposed mechanisms, such as viscous annihilation and amalgamation of opposite sign vortex pairs (Jimenez 1983; Rogers & Moser 1993), have not been observed in direct measurements, in principle there is no reason why they should not occur, providing the local circumstances, such as streamwise vortex strength and spacing for example, are appropriate.

The balance of experimental evidence suggests that the spanwise wavelength of the streamwise structures will increase, scaling approximately with the mixing-layer vorticity thickness. However, the increase will in general be nonlinear (stepwise) with the jumps coinciding with some pairing locations. An increase in spanwise wavelength has not been observed experimentally in the absence of spanwise roller pairings. However, it is quite clear that the spanwise scale does not have to increase at every spanwise roller pairing location, although it can, as shown by Huang & Ho (1990). This means that a constant ratio of spanwise to streamwise wavelength will not be maintained in general. As Rogers & Moser (1993) suggested, in practice, the details of the spanwise scale change will probably be determined by the incoming disturbance environment.

5. Conclusions

An experimental study designed to investigate spanwise scale changes of the streamwise vortical structure in a plane mixing layer has been completed. Detailed

three-dimensional phase-averaged measurements were obtained of the spanwise and streamwise vorticity in a forced mixing layer undergoing three pairings.

Approximately two pairs of streamwise vortices, with one pair distinctly stronger than the other, were observed in the measurement domain just downstream of the first spanwise vortex roll-up. The overall spanwise scale remained unchanged through the first two roller pairings. However, some 'readjustments' of the weaker streamwise structures were apparent. In particular, the weaker positive structure went through several cycles of splitting up into smaller structures and then remerging into a single one. In the meantime, the spanwise spacing between the stronger structures remained unchanged. During the third pairing, the weaker positive structure split for the last time, as it was displaced by the neighbouring structures of negative sign which then merged together. Hence, the overall spanwise scale was doubled downstream of the third roller pairing and the circulation of the merged structure was twice that of one of the original structures.

The present results clearly show that an increase in spanwise scale occurs during a (third) pairing of the spanwise rollers, but that an increase does not have to occur at every roller pairing. The obvious question is whether the next increase in spanwise scale will only occur after another three pairings. While this question can obviously not be answered using the present results, the balance of evidence suggests that this is not the case. For example, the mean streamwise vorticity measurements of Bell & Mehta (1992) show that the next scale change occurred at the estimated location of the next (fourth) pairing. As suggested by Rogers & Moser (1993), the exact form and evolution of the spanwise scale change are strongly dependent on the nature of the initial disturbance field. It is worth noting that a spanwise scale change is not observed when a uniform array (regular strength, size and spacing) of streamwise vortices is injected into a mixing layer (Bell & Mehta 1993).

Although the present measurements were obtained in an acoustically forced mixing layer, the details of the spanwise scale change in an unforced layer are not expected to be very different. In particular, the present results clearly show that the presence and location of the measured scale change is certainly not an artifact of the acoustic forcing.

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